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Dynamic Spectrum Sharing with Multiple Primary and Secondary Users

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***Abstract*— Dynamic spectrum sharing in cognitive radio networks can enhance flexibility, and as a result the efficiency of spectrum usage. In this paper, we address the problem of spectrum sharing in a cognitive radio network where multiple primary and secondary strategic-users are involved. In this scenario, primary users (PUs) would like to offer part of their spectrum to secondary users (SUs) to make extra revenue. PUs face a trade-off since the more spectrum that is being shared with SUs, the more Quality of Service (QoS) degradation their own service will suffer. SUs access the Internet through an Access Point (AP). They have to decide their best strategies by taking into account service satisfaction and payment lost. The profit of PUs and SUs is directly related to the bandwidth allocation and price charging through the AP. Considering this competitive relationship, we model the scenario as a noncooperative game, and analyze it by exploring the properties of Nash Equilibrium (NE) point. The simulation results support our theoretic analysis.**

I. Introduction

Dynamic spectrum sharing is a promising mode for reusing the underutilized spectrum, where spectrum is shared among primary (licensed) and secondary (unlicensed) users to improve spectrum flexibility and therefore efficiency. Spectrum marketing is an effective way to realize the spectrum sharing. Among all challenges to spectrum marketing, economical modeling is one major issue, and game theory [1] is widely used to model the behaviors of players (users).

The future dynamic spectrum sharing paradigm is most probably to be associated with multiple PUs and SUs. Users are rational and selfish and thus only care about their own payoff and always take best actions. In this paper, we consider the modeling of dynamic spectrum sharing in a Cognitive Radio (CR) network (single cell) with multiple strategic PUs and SUs as shown in Fig. 1. PUs have spectrum bands for their own primary services. They are willing to lease part of their bands to SUs for additional income. However, the more bandwidth PUs lease, the more QoS degradation their services will suffer. SUs make demands on the spectrum and access the Internet through an AP. The AP acts as a coordinator, buys spectrum usage from PUs, allocates it to SUs and charges them, such that the profits and strategies of the two-layer users are related. The motivation of AP is to bridge PUs and SUs, and even promote the traded volume in the market, but not to raise revenue.

We assume that AP has the control power over unit price for spectrum in the market. PUs bid for the amount of spectrum they are willing to lease and SUs bid the amount they would like to buy. The bidding is considered to be the "willingness" to trade, and the allocation by AP is according to the "willingness", which is a proportional allocation. We leverage game theory to model the competitive relationships: PUs leasing their spectrum and SUs purchasing spectrum. We demonstrate in this paper that, there exists at least one Nash Equilibrium point, and in fact a unique NE for most common cases. The analysis of efficiency shows that the NE is Social Optimal for one side of users and at least Pareto Optimal for another side. We also discuss how to set the price in the market to promote the traded volume. The simulation results confirm the existence and uniqueness of NE, the relationship between the price adjustment and the traded volume, and the high efficiency of NE.

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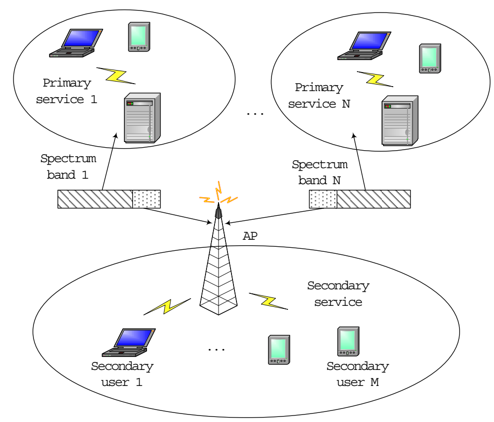

Fig. 1. Spectrum sharing model

The contribution of our work is a two fold. First, we propose a game model which involves multiple PUs and SUs, and enables every user to form a strategy. Second, we give theoretic proof about the unique NE point, its performance and simulation results to verify it.

The rest of this paper is organized as follows: In Section II, the related research is summarized. We describe the system model involving the function of AP and the utility functions of all users in Section III. In Section IV, we analyze the game and explore the properties of NE point(s). Simulation results are offered to confirm the theoretic analysis in Section V. Finally, Section VI concludes the paper.

II. Related Works

Dynamic spectrum sharing issues are discussed in many research efforts. In this section, we summarize the related works on economical modeling of dynamic spectrum sharing.

Some research can be characterized by the model involving one PU and multiple SUs. We further classify them according to the market mechanism. Some used the auction model, such as multi-unit sealed-bid auction in [2] and multi-unit second-price (Vickrey) auction in [3]. Some used linear/nonlinear pricing. In [4], Cournot game model was proposed to model the oligopoly market competition. Different from these, some mechanisms enabled exchanging secondary transmission power with spectrum accessing. In [5], Stackelberg game was used to model the spectrum leasing to cooperating secondary Ad Hoc networks involving one primary link and multiple SUs. In the scenario, the PU chose a group of SUs to relay for itself, and the chosen users got partial spectrum accessing rights in return. The authors in [6] studied a similar scenario by combining the payment mechanism to the original one.

There are works considering multiple PUs and one SU. For example, Bertrand game model was used to obtain the optimal pricing in an oligopoly market [7].

The model has been extended to multiple PUs and SUs in some recent works. In [8], a two-tier trading system with a spectrum broker, multiple providers and users was proposed. Spectrum was allocated to service providers by the winner determining sealed-bid knapsack auction mechanism. Service providers then served end users directly in a dynamic pricing game model. Our work is different from [8] because in our model, PUs just lease part of their spectrum to SUs and suffer QoS degradation to some extent, and SUs utilize the spectrum through AP. The research in [9] proposed a similar framework modeled as a two-stage Stackelberg game. Its main contribution was relating the two-stage strategic behaviors of PUs, while its drawback was that the model did not involve strategic SUs. However, both PUs and SUs are strategic users in our model. [10] modeled the PUs' and SUs' competition by non-cooperative game and evolutionary game respectively. The existence and uniqueness of the NE was proved and an iterative algorithm to obtain the NE was proposed. However, PUs just made equal spectrum allocation and ignored SUs' different spectrum requirements and abilities to pay. In our model, all the users are enabled to independently form strategy, and thus freely express their willingness.

III. System Model

In this section, we describe our system model and game theoretical formulation. N PUs and M SUs are players in the game, and an AP is a nonprofit coordinator as shown in Fig. 1. PUs decide the amount of bandwidth to lease and SUs decide the amount of bandwidth to buy. AP collects all the strategies and makes a proportional allocation. We model it as a noncooperative game where the competition is between individual participants.

PU P_i has its strategy b_i as a "bidding bandwidth" in the space $(0, \overline{b_i}]$, where b_i denotes the maximum bandwidth it is willing to lease and $\overline{b_i}$ is its total bandwidth amount. SU S_j has a strategy r_j in the space $(0, \overline{r_j}]$, which denotes its rate requirement. $\overline{r_j}$ is the constant up-bound rate, which depends on the capabilities of S_j and the AP. As S_j exclusively occupies the band (using FDMA), it and AP can transmit under any power level which does not exceed their capabilities without interference from others. So the rate requirement r_j can be translated into bandwidth requirement B_j with Shannon's Capacity equation:

$$r_j = B_j L_j$$

where $L_j = \log(1 + \gamma_j)$, and the γ_j is the SNR which is supposed to be a constant.

Table I. Index of key symbols

Symbol	meaning
b_i, r_j	strategy of P_i,S_j
$\overline{b_i}, \overline{r_j}$	strategy up-bound of P_i,S_j
$\widehat{b_i}, \widehat{r_j}$	allocated volume for P_i,S_j
$\widetilde{b_i}$	best strategy for P_i when $T_b \leq T_B$
$\widetilde{r_j}$	best strategy for S_j when $T_B \leq T_b$
$B_i, \overline{B_i}, \widehat{B_i}, \widetilde{B_i}$	$r_i, \overline{r_i}, \widehat{r_i}, \widetilde{r_i}$ divide $\log(1+\gamma_i)$ respectively
$T_b, \widetilde{T_b}, T_B, \widetilde{T_B}$	sum of b_i,$\widetilde{b_i}$,B_i,$\widetilde{B_i}$ respectively

A. Resource Allocation of AP

Initially, AP decides the price p of unit spectrum, and informs all users. Then it collects bidding $(b_i, r_j, i = 1, \ldots, N, j = 1, \ldots, M)$ from all users, proportionally divides the traded volume D among them, and sends its allocation result $(\widehat{b_i}, \widehat{r_j}, i = 1, \ldots, N, j = 1, \ldots, M)$ to them.

$$D = \min\{\sum_{i=1}^{N} b_i, \sum_{j=1}^{M} B_j\} = \min\{\sum_{i=1}^{N} b_i, \sum_{j=1}^{M} \frac{r_j}{L_j}\}$$

We assume a proportional allocation rule in which allocation to a user is proportional to its bidding. This type of allocation rule has been studied in a wide range of applications, including network resource allocation [11] [12].

$$\widehat{b_i} = b_i \frac{D}{\sum_{j=1}^{N} b_j}, \ \ \widehat{B_i} = B_i \frac{D}{\sum_{j=1}^{M} B_j}, \ \ \widehat{r_i} = \widehat{B_i} L_i$$

That is to satisfy the users whose bidding sum is relatively small, and to divide D fairly among the other users. To do this, we suppose that bandwidth can be divided and integrated arbitrarily.

B. Primary Service

To model the utility function for P_i, we consider three factors: the primary service revenue R_i, the spectrum leasing incoming I_i and the QoS degradation cost C_i as in [7]. Since the PUs lease their in-use bandwidth to SUs, the revenue and QoS degradation of primary services should be considered naturally.

R_i is generated by utilizing P_i's all spectrum $\overline{b_i}$. Both R_i and $\overline{b_i}$ are assumed to be constants.

Spectrum leasing incoming I_i is based on linearly pricing: $I_i = p\widehat{b_i}$, where p is the price for unit spectrum block in frequency and time domain.

To model the cost due to QoS degradation C_i, we assume it is monotonously increasing with respect to $\widehat{b_i}$. We take the detailed form of C_i $(0 \leq C_i \leq R_i)$ as quadratic [7]:

$$C_i = R_i(\widehat{b_i}/\overline{b_i})^2$$

The utility function for P_i combines the above three factors:

$$U_{P_i} = U_{P_i}(\mathbf{b}, \mathbf{r}) = R_i + p\widehat{b_i} - R_i(\widehat{b_i}/\overline{b_i})^2 \quad (1)$$

C. Secondary Service

To model the utility function for S_j, we have to consider two factors: QoS satisfaction δ_j and the payment to AP β_j.

We assume the data traffic of SUs is elastic. QoS satisfaction of the elastic traffic satisfies the logarithmic form [8]:

$$\delta_j = \theta_j \log(1 + \widehat{r_j}) = \theta_j \log(1 + \widehat{B_j} L_j)$$

where θ_i is a positive constant and indicates the relative importance of the QoS satisfaction.

The payment to AP also has a linear form: $\beta_j = p\widehat{B_j}$

Utility function for S_j is

$$U_{S_j} = U_{S_j}(\mathbf{b}, \mathbf{r}) = \theta_j \log(1 + \widehat{B_j} L_j) - p\widehat{B_j} \quad (2)$$

IV. The Analysis of Nash Equilibrium point(s)

In this section, we prove the general existence of an NE point, and its uniqueness under some conditions. The efficiency of NE is also analyzed, and a condition to maximize the traded spectrum volume is discussed afterwards.

A. Nash Equilibrium (NE) of the game

We first briefly introduce the concept of NE and then explore some properties of utility functions to get the best response for all users. The fixed-point theorem is utilized to get the existence of NE point(s). Finally by reduction to absurdity, the uniqueness of NE is proved.

1) Definition of NE and best response (strategy):

Definition 1: A vector $\mathbf{P} = (b_1, \ldots, b_N, r_1, \ldots, r_M)$ is a NE of the game $G = \{N + M, \{P_k\}, \{U_k(\cdot)\}\}$ if for every user k, $U_k(p_k, \mathbf{p}_{-k}) \geq U_k(p_k^{'}, \mathbf{p}_{-k})$, for all $p_k^{'} \in P_k$.

In other words, each user cannot unilaterally increase its own utility in the NE state. Because each user k has taken the best response which can maximize its own utility, given the others' strategies $\mathbf{p}_{-k}$. NE is usually obtained by using the best response functions. Mathematically, we differentiate the utility functions to find the best response for users. For simplicity, we define $T_b = \sum_{j=1}^{N} b_j$ and $T_B = \sum_{j=1}^{M} B_j$. The first derivative of U_{P_i} with b_i is

$$\frac{\partial U_{P_i}}{\partial b_i} = \begin{cases} p - \dfrac{2R_i b_i}{\overline{b_i}^2} & \text{if } T_b \leq T_B \\ (p - \dfrac{2R_i b_i T_B}{\overline{b_i}^2 T_b})\dfrac{T_B(\sum_{k\neq i} b_k)}{T_b^2} & \text{else} \end{cases}$$

It is easy to get the best response for P_i:

$$b_i = \begin{cases} \min\{\dfrac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\} & \text{if } T_b \leq T_B \\ \min\{\dfrac{p\overline{b_i}^2(T_b - b_i)}{2R_i T_B - p\overline{b_i}^2}, \overline{b_i}\} & \text{elseif } 2R_i T_B > p\overline{b_i}^2 \\ \overline{b_i} & \text{else} \end{cases} \quad (3)$$

Similarly the best response for S_j is

$$r_j = \begin{cases} \min\{\dfrac{\theta_j L_j}{p} - 1, \overline{r_j}\} & \text{if } T_b \geq T_B \\ \min\{\dfrac{(T_B - B_j)(\theta_j L_j - p)}{pT_b - \theta_j + \frac{p}{L_j}}, \overline{r_j}\} & \text{elseif } T_b > \dfrac{\theta_j}{p} - \dfrac{1}{L_j} \\ \overline{r_j} & \text{else} \end{cases} \quad (4)$$

Here we assume that $\theta_j L_j > p$, because otherwise S_j's best strategy is $r_j \to 0$, which means that S_j does not have incentive to anticipate in the spectrum market.

2) Existence of NE point(s):

Next, we will prove the general existence of NE point(s).

Lemma 1: For each user, its utility function is quasi-concave in its own strategy.

Proof: Let $\widetilde{b_i} = \min\{\frac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\}$ and $\widetilde{B_j} = \min\{\frac{\theta_j}{p} - \frac{1}{L_j}, \frac{\overline{r_j}}{L_j}\}$. For P_i, if $\sum_{k\neq i} b_k \geq T_B$, then U_{P_i} is concave, thus quasi-concave. Otherwise, let $x = T_B - \sum_{k\neq i} b_k > 0$. If $x \leq \widetilde{b_i}$, U_{P_i} increases in the interval $(0, x]$ and $(x, y]$, and decrease in the interval (y, ∞), where y is the best strategy of P_i when $T_b > T_B$. Also we note that U_{P_i} is continuous at point x. If $\overline{b_i} < y$, U_{P_i} is strictly increasing, otherwise it is first increasing and then decreasing. So U_{P_i} is quasi-concave, because by definition a function is quasi-concave if its upper contour sets are convex sets. Similarly, if $x > \widetilde{b_i}$, U_{P_i} is also quasi-concave.

Up to now, we have proven that U_{P_i} is quasi-concave. Similarly, it can be proved that the utility of S_j is also quasi-concave. ■

Theorem 1: There exists at least one NE point for the game.

Proof: Kakutani's fixed-point theorem guarantees that a Nash Equilibrium exists in the game $G = \{N + M, \{P_k\}, \{U_k(\cdot)\}\}$ if for all $k = 1, \ldots, N + M$,

1) The strategy space P_k is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^N$
2) Every utility function $U_k(\mathbf{P})$ is continuous in strategy profile $\mathbf{P}$ and quasi-concave in its own strategy p_k.

It is easy to check that the strategy space, either $(0, \overline{b_i}]$ or $(0, \overline{r_i}]$, satisfies the condition 1. Although the utility functions contain the minimum function, they are still continuous in $\mathbf{P}$. According to Lemma 1, the utility functions are all quasi-concave. ■

3) Uniqueness of NE under some conditions:

We will discuss the uniqueness of NE under some condition, and also the case when the condition is not satisfied.

Claim 1: In NE state, either P_i is satisfied with its bidding amount $b_i = \widetilde{b_i}$ (for all i), or S_j is satisfied with its bidding amount $r_j = \widetilde{r_j}$ (for all j).

Proof: Suppose there are P_i and S_j not satisfied with their bidding in the NE state. If now $T_b < T_B$, then P_i must be satisfied with $b_i = \widehat{b_i}$, which also equals P_i's best strategy $\widetilde{b_i}$ (otherwise it is not an NE state, as P_i can improve its utility unilaterally). If $T_b \geq T_B$, then S_j must be satisfied with $r_j = \widehat{r_j}$, which also equals to S_j's best strategy $\widetilde{r_j} = \widetilde{B_j}\log(1+\gamma_j)$. This is a contradiction. ■

Theorem 2: The game has a unique NE point when $\widetilde{T_b} \neq \widetilde{T_B}$, where $\widetilde{T_b} = \sum_{i=1}^{N} \widetilde{b_i}$ and $\widetilde{T_B} = \sum_{i=1}^{M} \widetilde{B_i}$.

Proof: Without loss of generality, suppose $\widetilde{T_b} < \widetilde{T_B}$. If there are two NE points N_1 and N_2, they must satisfy $b_i = \widehat{b_i} = \widetilde{b_i}$ for all $i = 1, \ldots, N$, according to Claim 1. So N_1 can only be different from N_2 at some S_i.

S_i' strategy is B_{i_1} in N_1, and B_{i_2} in N_2. Without loss of generality, suppose $\overline{B_i} \geq B_{i_1} > B_{i_2}$. Then in N_2, $\widehat{B_{i_2}} = \widetilde{B_i}$, otherwise S_i can unilaterally increase its utility so N_2 is not NE. Compared to N_1, S_i decreases its strategy and keeps its utility non-decreasing, which means $\frac{\sum_{j\neq i} B_{j_2}}{\sum_{j\neq i} B_{j_1}} \leq \frac{B_{i_2}}{B_{i_1}}$.

If $\frac{B_{j_2}}{B_{j_1}} = \frac{B_{i_2}}{B_{i_1}}$, for all $j \neq i$, then N_2 is not NE, because in N_1, there must exist some S_k, such that $\overline{B_k} = B_{k_1} > \widehat{B_{k_1}}$ and $\widetilde{B_k} > \widehat{B_{k_1}}$. In N_2, S_k can unilaterally increase its utility, so N_2 is not NE.

Otherwise, there exists some S_k, such that $\frac{B_{k_2}}{B_{k_1}} \leq \frac{B_{j_2}}{B_{j_1}}$ for all $j = 1, \ldots, M$ and $\frac{B_{k_2}}{B_{k_1}} < \frac{B_{j_2}}{B_{j_1}}$ for some j, then we have $\frac{\sum_{j\neq k} B_{j_2}}{\sum_{j\neq k} B_{j_1}} > \frac{B_{k_2}}{B_{k_1}}$. So S_k can unilaterally increase its utility in N_2. N_2 is not NE.

The contradiction means that there cannot be two NE points in this case. ■

For the case where $\widetilde{T_b} = \widetilde{T_B}$, there may exist infinite number of NE points. Let $\{\mathbf{b}_0, \mathbf{r}_0\}$ denotes the NE point where $b_i = \widehat{b_i} = \widetilde{b_i}$ and $r_j = \widehat{r_j} = \widetilde{r_j}$ for all i,j. Then the strategy profile $\{\mathbf{b}_i, \mathbf{r}_j\}$ is NE if it satisfies that

$$\mathbf{b}_i = \mathbf{b}_0 \text{ and } \mathbf{r}_j = \beta\mathbf{r}_0$$

where $\beta \geq 1$, $\beta\mathbf{r}_0 \leq \{\overline{r_1}, \ldots, \overline{r_M}\}$, or

$$\mathbf{b}_i = \beta\mathbf{b}_0 \text{ and } \mathbf{r}_j = \mathbf{r}_0$$

where $\beta \geq 1$, $\beta\mathbf{b}_0 \leq \{\overline{b_1}, \ldots, \overline{b_N}\}$.

However, all possible NE points in fact lead to the same allocation result, which means the NE points are semi-scale-free. What is more, if every user knows complete information about other users, everyone can predict that it can achieve its best utility, then it can take the strategy $b_i = \widetilde{b_i}$ or $r_i = \widetilde{r_i}$. In this case, the game will converge to unique NE state in only one step.

B. Efficiency of the NE

To discuss the efficiency, we consider the total utility of PUs $\sum_{i=1}^{N} U_{P_i}$ and that of SUs $\sum_{i=1}^{M} U_{S_i}$ in the network. In this paper, the terminology "Social Optimal" means the possibly maximum sum of utilities of referred users, and "Pareto Optimal" is defined for referred users as follows.

Definition 2: A strategy profile $\mathbf{P}$ Pareto dominates another $\mathbf{P}'$ if, for all $k \in N+M$, $U_k(\mathbf{P}) \geq U_k(\mathbf{P}')$ and for some $j \in N+M$, $U_j(\mathbf{P}) > U_j(\mathbf{P}')$. Furthermore, $\mathbf{P}^*$ is Pareto Optimal (efficient) if it is not Pareto dominated by any other $\mathbf{P}$.

Theorem 3: In the NE state, if $\widetilde{T_b} = \widetilde{T_B}$, all the NE points are Social Optimal; if $\widetilde{T_b} < \widetilde{T_B}$, $\{b_1, \ldots, b_N\}$ is Social Optimal for PUs and $\{r_1, \ldots, r_M\}$ is Pareto Optimal for SUs; if $\widetilde{T_b} > \widetilde{T_B}$, $\{b_1, \ldots, b_N\}$ is Pareto Optimal for PUs and $\{r_1, \ldots, r_M\}$ is Social Optimal for SUs.

Proof: If $\widetilde{T_b} = \widetilde{T_B}$, then every user will be allocated $\widetilde{b_i}$ or $\widetilde{r_i}$. So $\{b_1, \ldots, b_N, r_1, \ldots, r_M\}$ is Social Optimal.

If $\widetilde{T_b} < \widetilde{T_B}$, every P_i can be satisfied with strategy $\widehat{b_i} = b_i = \widetilde{b_i}$, so $\{b_1, \ldots, b_N\}$ is Social Optimal. There must be at least one SU with strategy $\widehat{r_i} < \widetilde{r_i}$, otherwise it will contradict $\widetilde{T_b} < \widetilde{T_B}$, and SUs completely divide $\widetilde{T_b}$. If any SU wants to increase its utility, it must reduce another user's utility. By Definition 2, it is Pareto Optimal. The case $\widetilde{T_b} > \widetilde{T_B}$ is similar. ■

C. Adjustment of parameter p

The above discussion is based on a fixed p. In fact, AP may adjust p according to some principle, such as to maximize the traded volume $D = \min\{T_b, T_B\}$. For PUs, the total strategy T_b is non-decreasing with p. For SUs, the total strategy is T_B, which is non-increasing with p. Besides, we have

$$T_b \leq \sum_{i=1}^{N} \min\{\frac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\}, \;\; T_B \leq \sum_{j=1}^{M} \min\{\frac{\theta_j}{p} - \frac{1}{L_j}, \frac{\overline{r_j}}{L_j}\}$$

So AP can adjust p to satisfy the following:

$$\sum_{i=1}^{N} \min\{\frac{p\overline{b_i}^2}{2R_i}, \overline{b_i}\} = \sum_{j=1}^{M} \min\{\frac{\theta_j}{p} - \frac{1}{L_j}, \frac{\overline{r_j}}{L_j}\}$$

to maximize the traded volume. The physical meaning of this formula is that to make $\widetilde{T_b}$ and $\widetilde{T_B}$ match. It is difficult to get the optimal p in the above formula directly, but by iterations the p can gradually approach to the optimal value.

V. Simulation Results

In this section, we will conduct simulations to verify the previous theoretic statements. We consider a cognitive radio environment where $N = 3$, $M = 7$, $p = 3$, and randomly select $R_i \in [500, 1000]$, $\overline{b_i} \in [50, 100]$, $\theta_i \in [5, 10]$, $\gamma_i \in [100, 200]$ and $\overline{r_i} \in [100, 200]$. The following results are based on the same initial parameters.

Fig. 2 supports the claim that the existence and uniqueness of NE point. The NE state achieved by iterations coincides with theoretic one, no matter what information is known to every users. Besides, we can observe that T_b is much larger than T_B. In fact, PUs all take their strategy up-bounds to compete for more traded amount and no PU is fully satisfied since $\widetilde{T_b} > \widetilde{T_B}$. However, all SUs are satisfied with $B_i = \widehat{B_i} = \widetilde{B_i}$.

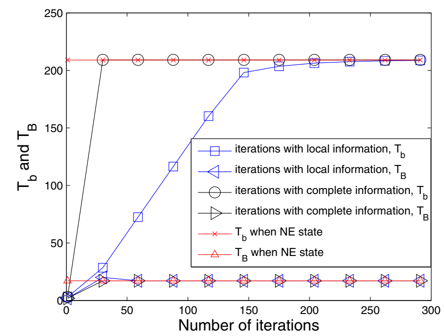

Fig. 2. Existence and uniqueness of the NE point

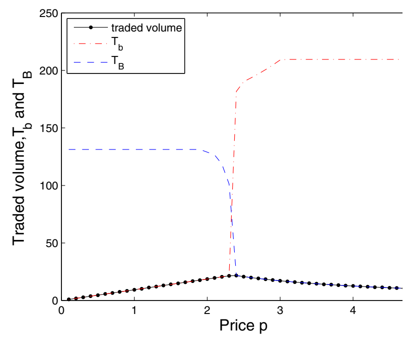

Fig. 3. Adjustment of price p

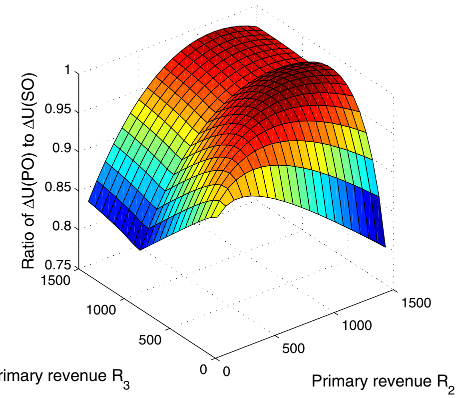

Fig. 4. Ratio of $\Delta U_p(PO)$ to $\Delta U_p(SO)$ when adjusting R_2, R_3

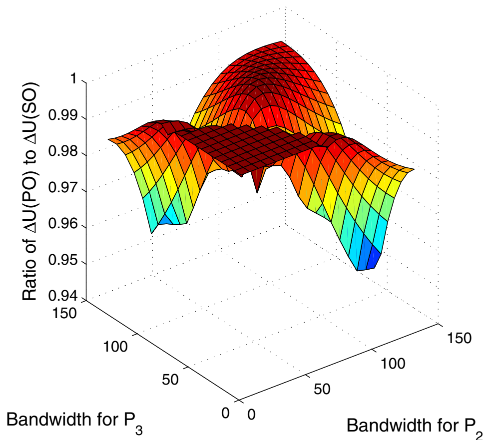

Fig. 5. Ratio of $\Delta U_p(PO)$ to $\Delta U_p(SO)$ when adjusting $\overline{b_2}$, $\overline{b_3}$

Fig. 3 verifies the claim that AP can adjust the price p to maximize the traded volume in the market, and make the potential best supply $\widetilde{T_b}$ and demand $\widetilde{T_B}$ matched at the same time. In the market, $p > 2.4$ is a high price which leads to $T_B \leq T_b$, so the traded volume will be constricted by T_B. By adjusting p, the bottleneck is eliminated and the supply matches demand.

To evaluate the efficiency, we adjust some parameters for PUs based on the former setting. In Fig. 4 and Fig. 5, we present the ratio of $\Delta U_p(PO)$ to $\Delta U_p(SO)$, where $\Delta U_p(PO)$ is the the improvement of the total utility of PUs under Pareto Optimal and $\Delta U_p(SO)$ is the one under Social Optimal. In Fig. 4, the primary service revenue R_1 and all bandwidth up-bounds are fixed, while R_2, R_3 are adjusted within the interval $(0, 2R_1]$. The ratio is relatively lower when R_2 (R_3) is small. That is because P_2 (P_3) can be allocated more bandwidth while suffering little QoS degradation in this case of Social Optimal. So the gap between Pareto Optimal and Social Optimal profiles is increased. In Fig. 5, the bandwidth up-bound $\overline{b_1}$ and all primary service revenues are fixed, while $\overline{b_2}$, $\overline{b_3}$ are adjusted within the interval $(0, 2\overline{b_1}]$. Generally speaking, the ratio is no less than 95%.

VI. Conclusions

In this paper, we have proposed a game-theoretic model to solve the problem of dynamic spectrum sharing with multiple strategic PUs and SUs. By modeling the scenario as a noncooperative game, we prove the general existence of NE and then its uniqueness under some conditions. The performance of NE point is considerably good, which is at least Pareto Optimal, or even Social Optimal. We also discuss the adjustment of price by AP to maximize the traded volume. The simulation results support our theoretic analysis.

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